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by

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I. INTRODUCTION

The purpose of this dissertation is to document research in the area of electrical wave filters, electrical circuits which discriminate or separate electrical waves on the basis of their frequencies of oscillation. The research focuses on passive 3-port filters, which have one input port and two output ports. The goal of this research was to discover or invent 3-port filters which have the Chebyshev or elliptic frequency response characteristic. Previous work by other researchers has resulted in 3-port filters which have maximally-flat or Butterworth frequency response characteristics. The research presented in this dissertation was intended to be an extension of this previous work.

This research stays within the confines of passive, physically realizable filters, which can be built using physical components: resistors, capacitors and inductors. Research in this field has dwindled in recent years, due in large part to the common use of digital computers and digital signal processing technology. The use of digital signal processing theory in conjunction with a digital computer allows the designer to create almost any frequency response characteristic, since the design is not restricted to mathematical functions which are based on real, positive coefficients given by the physical components. The challenge presented by passive filters is to find the conditions under which the desired result can be achieved without the benefit of the relaxed constraints offered by digital signal processing. Even with the rapid advancement of digital signal processing as the method of choice in most modern electronic systems, passive filtering still has uses in the higher frequency design areas. Digital computers are not yet fast enough to perform the operations necessary to filter signals at frequencies higher than a few megahertz, and these filters must be designed using the traditional methods. Therefore, research in this area is still warranted for improved electrical systems, as well as the extension of prior theory.

II. STATEMENT OF THE PROBLEM

A. 3-Port Filters

1. Description

Three-port filters, as their name implies, have three pairs of terminals, each terminal pair acting as a port. A terminal pair acts as a port if all of the current entering one of the terminals in the pair is returned through the other terminal of the pair. Each port in the 3-port filter may be terminated with a load impedance and a voltage source or a current source. Voltage sources are generally connected in series with the load impedance (Thevenin equivalent), while current sources are connected in parallel with the impedance (Norton equivalent). An example of a very general 3-port filter is shown in figure 2.1. This figure shows the two types of load and source connections as well as the port naming conventions which will be used throughout this dissertation. All three ports are not required to possess a source. At least one of the ports usually is driven by a source, and the connection at the remaining ports is arbitrary. This research is focused on 3-port filters which have one port driven with a voltage source and a series resistance. The remaining ports are not driven, but are terminated with a load resistor. Since only one port is

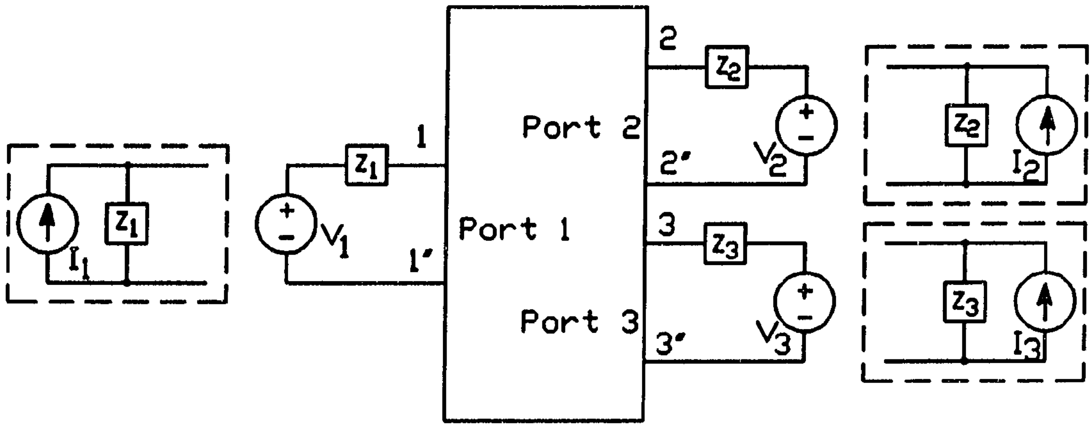

Figure 2.1. 3-port filter with load terminations

driven, power generally flows from the driven port to the two ports which are resistively terminated. Thus, the two separate pathways, from the driven port to the two undriven ports, constitute a 3-port connection which is made up of two 2-port filters connected in parallel at the input ports. Figure 2.2 shows a simplified view of this connection. Since the 3-ports can be viewed as a parallel connection of 2-ports, the 3-ports are commonly called filter pairs.

2. Uses and interest

Filter pairs are useful whenever there is a need to split the frequency spectrum of the input signal into two disparate output frequency spectra. Usually, and specifically in this

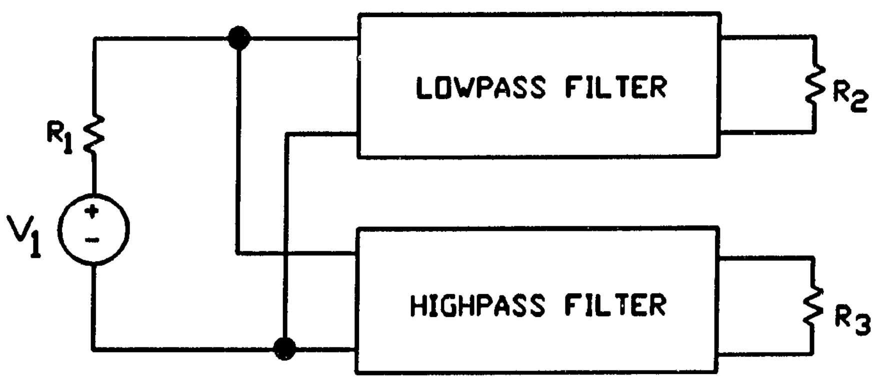

Figure 2.2. Simplified diagram of 3-port filter

research, one of the two filters will pass the frequencies below a predetermined frequency, and the other filter will pass the frequencies above the frequency. The former filter is termed a lowpass filter, and the latter is termed a highpass filter. This frequency separation is shown for a Butterworth filter pair in figure 2.3. The frequency response of the output of the lowpass filter and the highpass filter are shown, with the lowpass response on the left side of the graph.

Filter pairs are used in telephone systems, wherein several filter pairs are used to form a filter group. These filter groups are used to separate a single-channel line into a multi-channel line to carry several conversations

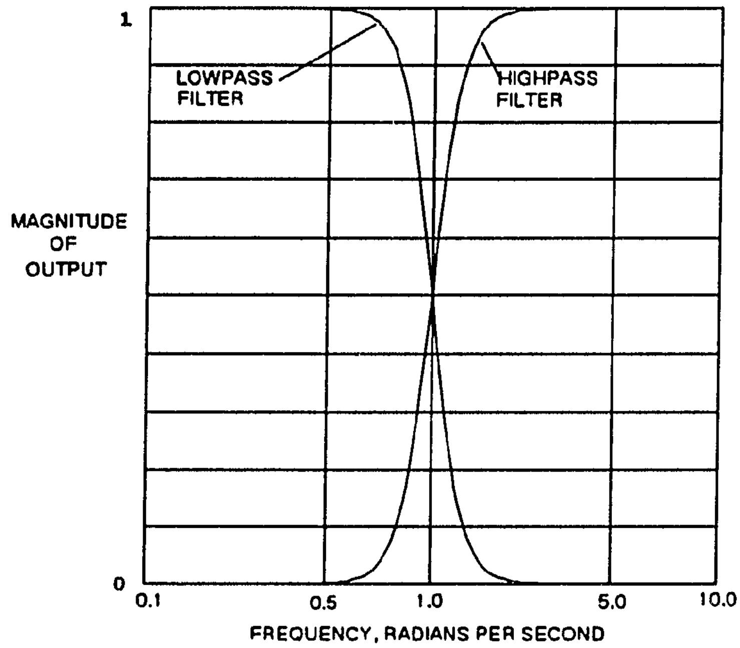

Figure 2.3. Plot of transducer power gain vs. frequency of Butterworth filter pair to show frequency separation caused by lowpass and highpass filters

simultaneously. The entire North American telephone network, as well as most of the systems in the world, is constructed in this manner.

3. Butterworth, Chebyshev and elliptic characteristics

The Butterworth, Chebyshev and elliptic filters are all approximations to the ideal, or "brickwall", filter. The frequency response of the ideal filter is flat with no attenuation until a predetermined frequency, and is equal to zero beyond that frequency. The ideal frequency response is impossible to achieve, and the three methods listed above are the three most-often discussed methods for obtaining approximations to the ideal response.

A convenient method of characterizing filters is by the form of their transducer power gain, $|\tau(\omega)|^2$. The transducer power gain is the ratio of the average power delivered to the load to the maximum average power available at the source. The maximum available power is a function of the source and load impedances of the filter, and the transducer power gain expression automatically normalizes the effect of the load by using the ratio of delivered power to available power. The Butterworth, Chebyshev and elliptic filters share a common mathematical structure for the transducer power gain:

$$|\tau(\omega)|^2 = \frac{H_0}{1 + \epsilon^2 P_n^2(\omega)} \qquad (2.1)$$

where H_0 represents the magnitude of the peak of the filter response, ϵ is a factor which controls the magnitude of the ripple in the filter response, and $P_n(\omega)$ is a polynomial or rational function that dictates the basic response of the filter. The characteristics of the three types of filters are described in detail in the literature (21, ch.2), and are summarized in the following sections.

Butterworth filters: The Butterworth filter is characterized as having no ripple in its frequency response characteristic. It is also referred to as having a "maximally flat" characteristic, since for an nth-order filter, the first n-1 derivatives of magnitude with respect to frequency evaluate to zero at the extreme passband frequency. The passband for a Butterworth lowpass filter is the range from zero frequency (DC) to the cutoff frequency (frequency at which the output power has decreased to 50% of the maximum). The passband for the highpass filter ranges from the cutoff frequency to infinity. For a 3rd-order Butterworth lowpass filter, the first 2 derivatives of $|\tau(\omega)|$ at zero frequency (DC) evaluate to zero. The frequency response characteristic appears very flat at DC, and decreases monotonically with frequency. Since there can be no ripple in the Butterworth characteristic, the ripple factor, ϵ, is set to 1. The approximation polynomial, $P_n^2(s)$, is = s^{2n}. The Butterworth transducer power gain is then:

$$|\tau(\omega)|^2_{Butterworth} = \frac{H_0}{1 + \omega^{2n}} \tag{2.2}$$

When the sinusoidal steady-state response of a filter is studied, the frequency variable, s, is restricted to the real-frequency, or imaginary, axis of the s-plane. This is because the Laplace transform of the sinusoid contains only real-frequency poles. Since the Laplace transform of the driving function contains only real-frequency components, the frequency variable used in the sinusoidal analysis needs no real parts. The substitution s = jω is made in the network functions to allow sinusoidal analysis. When this substitution is made in (2.2), the result is:

$$\tau(s)\tau(-s)\big|_{s=j\omega} = \frac{H_0}{1 + (-1)^n s^{2n}} \qquad (2.3)$$

The first response in figure 2.4 shows the Butterworth magnitude response. It can be seen to decrease monotonically with frequency.

Chebyshev filters: The Chebyshev filter is characterized as having ripples of equal magnitude in the passband only. The ripples are variations in the passband magnitude, and are the direct result of using a Chebyshev polynomial of the first kind as the approximation polynomial. The second response in figure 2.4 shows the passband ripple characteristic. The Chebyshev polynomial is designated $C_n(\omega)$, and is defined as:

$$C_n(\omega) = \cos(n \cos^{-1} \omega) \text{ for } 0 \leq \omega \leq 1 \qquad (2.4)$$
$$C_n(\omega) = \cosh(n \cosh^{-1} \omega) \quad \text{for } \omega \geq 1.$$

The ripple factor, ϵ, determines the degree to which the response can vary in the passband. A value of $\epsilon = 0.34931$, for example, yields a ripple magnitude of 0.5 decibels, which is the same as 5.6% peak-to-valley variation in voltage or current. The transducer power gain of the Chebyshev filter is then:

$$|\tau(\omega)|^2_{\text{Chebyshev}} = \frac{H_0}{1 + \epsilon^2 C_n^2(\omega)} \qquad (2.5)$$

When the frequency substitution $s = j\omega$ is made to allow sinusoidal response analysis, the Chebyshev transducer power gain becomes:

$$\tau(s)\tau(-s)\big|_{s=j\omega} = \frac{H_0}{1 + \epsilon^2 C_n^2(-js)} \qquad (2.6)$$

Elliptic filters: The elliptic filter was originally derived by Cauer, and is sometimes referred to as a Cauer (15) filter. The elliptic frequency response is characterized as having ripples of equal magnitude in its passband as well as in its stopband. The stopband in the elliptic filter case is defined as the region beyond the frequency at which the magnitude of the response has decreased to the stopband ripple value. The stopband does not start where the passband ends. The region between the passband and the stopband for the elliptic filter is referred to as the transition region. The magnitude of the ripples in the passband are not, in general, equal to those in the stopband. The third response in figure 2.4 shows the elliptic response.

The elliptic frequency response characteristic is obtained as a consequence of using a rational function, designated $F_n(\omega)$, as the approximation polynomial. The rational function is a Chebyshev rational function, and is calculated from the Jacobian elliptic integral functions. The theory of these functions is complex, and the reader is referred to Chen (17, ch. 3). In general, however, the approximating function, $F(\omega)$, can be written as:

$$F_n(\omega) = \mathrm{sn}\left[\frac{nK_1}{K}\mathrm{sn}^{-1}(\omega,k),k_1\right] \tag{2.7}$$

for n even, and

$$F_n(\omega) = \mathrm{sn}\left[K_1 + \frac{nK_1}{K}\mathrm{sn}^{-1}(\omega,k),k_1\right] \tag{2.8}$$

for n odd. The sn and sn^{-1} terms are the natural and inverse forms of the elliptic sine function, respectively. The terms K_1 and K are defined by:

$$K_1 = K(k_1) = F(k_1, \pi/2) \qquad (2.9)$$

$$K = K(k) = F(k, \pi/2) \qquad (2.10)$$

where $F(x,\pi)$ is the complete elliptic integral of the first kind of modulus x. The values for this function are tabulated with interpolation instructions in Abramowitz and Stegun (1). The parameters k and k_1 are the steepness factor of the transition region, and the gain constant, respectively. The transducer power gain for the elliptic filter is then:

$$|\tau(\omega)|^2_{elliptic} = \frac{H_0}{1 + \epsilon^2 F_n^2(\omega)} . \qquad (2.11)$$

The elliptic magnitude response for real-frequency analysis is:

$$\tau(s)\tau(-s)|_{s=j\omega} = \frac{H_0}{1 + \epsilon^2 F_n^2(-js)} . \qquad (2.12)$$

Figure 2.4 shows frequency response characteristics for the transducer power gains for the Butterworth, Chebyshev and elliptic filters. It can be seen that the transition region steepness for the elliptic filter is greater than that for the Chebyshev , and the same is true for the Chebyshev when compared to the Butterworth. The ripples in the passband of the Chebyshev and elliptic characteristics, and in the stopband of the elliptic characteristic are evident.

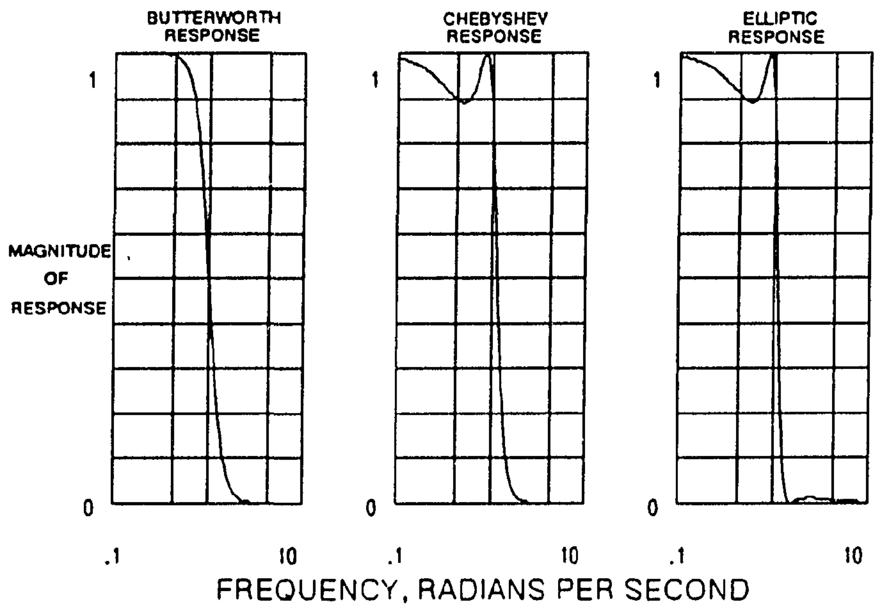

Figure 2.4. Frequency response characteristics for three classic filter types

Each of the filter responses can be chosen for a particular application based on their notable characteristics. Generally, passband ripples are the deciding factor between the Butterworth and Chebyshev (or elliptic) characteristic. If a smooth response is desired, the Butterworth response is chosen. As previously mentioned, the Chebyshev and elliptic responses both have sharper attenuation characteristics in the transition region between the passband and the stopband. This fact is important in some applications, especially in those applications in which efficient space usage is important. In such applications, a filter which develops the greatest attenuation with the fewest components is most valuable. These are the applications in which the Chebyshev and elliptic characteristics are important, and the central reason that this research was performed. It is important to have the Chebyshev and elliptic theory be up-to-date with the Butterworth theory, so that filter designers always have a choice of sharper transition filters than the Butterworth filters.

B. Prior work in Filter Pairs

1. Constant resistance filters

Norton (23) originally broke ground on filter pairs in his 1937 article on constant resistance filters. Constant resistance filters are characterized as having an input impedance which does not vary with frequency. Examples of filter structure and associated element values which yield constant input impedance filters are shown in figure 2.5. In the cases cited by Norton, each had a constant, real input impedance (input resistance). A natural consequence of

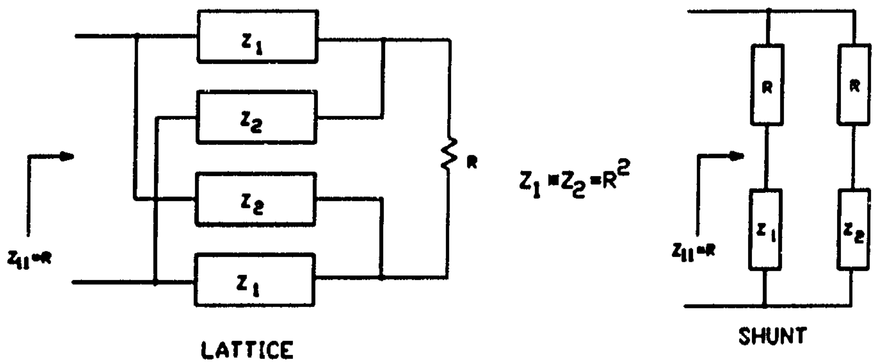

Figure 2.5. Examples of filter structures with constant input resistance

constant resistance 2-ports is that when two or more of them are connected in parallel at the input port, the resulting multiport network has again a constant input resistance. By connecting a lowpass constant resistance 2-port in parallel with a highpass constant resistance 2-port which has the same input resistance, Norton designed the first constant resistance filter pair. These filter pairs were used in a variety of applications in the telephone system. Since the entire telephone system is designed to have a characteristic impedance of 600 ohms, the filters designed by Norton were impedance-scaled to 600 ohms and used in the system directly. Another natural consequence of the constant resistance filter pairs is that the attenuation at the crossover frequency, which is the frequency at which the outputs of both filters are equal, is necessarily 3 dB. In systems which require the isolation between the two outputs to be greater than 3 dB, the constant resistance filters are not a good choice, and an alternate must be chosen. The alternate lies in the area of nonconstant impedance filters.

2. Butterworth filter pairs

As a means of avoiding the problem of lack of separation at the crossover frequency, the advantage of constant input resistance had to be abandoned. The convenience added to the design of whole systems of filters by the constant input impedance had to be given up in favor of filters which were not constrained by the input resistance characteristics. The result was a set of filters which had the Butterworth frequency response characteristic, but which also had input impedances which varied with frequency. Belevitch (9) and Zhu (26) worked independently in this area, and discovered different ways of designing Butterworth filter pairs.

C. Extension of Prior Work

1. Chebyshev and elliptic responses

Filter theory texts invariably include complete developments for the Butterworth, Chebyshev and elliptic approximations to the ideal lowpass filter. As a result, these three approximation techniques have become the "classic" filter techniques. Many tables of element values exist for the prototype versions of all three of these techniques (20). When a new method or variation is discovered for one of the classic filter approximations, comparable results are usually developed for the other approximations. The work by Belevitch and Chen covered only the Butterworth approximation technique. It was left to other researchers to develop the nonconstant impedance filter pairs for the Chebyshev and elliptic cases. The goal undertaken in this research was to show that an approximation to the Chebyshev and elliptic frequency responses could be achieved, and to show a means of designing the filter pairs with passive elements.

2. Complementary filter pairs

In order to limit the scope of the research to permit a reasonable time frame for completion, the research concentrated in the area of complementary filter pairs. In this arrangement, the lowpass and highpass filter transfer functions are reciprocal functions of frequency, meaning that if the lowpass filter transfer function is $F(s)$, where $s=\sigma+j\omega$ is the complex frequency variable, the transfer function of the highpass filter is given by $F(1/s)$.

This assumption also restricts the research to filters of equal order. For example, if the lowpass filter is third-order, the highpass filter mate is also third-order. Further work can be done to investigate the effects of uneven order on the frequency response of the filter pairs.

D. Synthesis of 3-port Filter Pairs

1. Current practice

The primary goal of the research was to develop a method for expressing the Chebyshev and elliptic filter pairs in some form of network function, such as a driving-point impedance function, or as a transfer function. The prior work used the scattering parameters as the means of specifying the characteristics of the filter pairs, so the same method was chosen for use in this research. A method was developed which describes the transmission and reflection characteristics of the filter pairs. Once these characteristics are known, the filter pair must be realized, which means it must be created from passive elements, as mentioned earlier.

The 3-port networks are characterized by their scattering parameters, which are the reflection and transmission coefficients for each of the three ports. The method used to synthesize the 3-ports can use either the reflection coefficient or the transmission coefficient as its starting point.

When the reflection coefficient of the 3-port is used as a starting point, a simple relationship yields the driving-point input admittance from the reflection coefficient:

$$Y_{in} = \frac{1 - S_{11}(s)}{1 + S_{11}(s)} \tag{2.13}$$

where S_{11} is the reflection coefficient and Y_{in} is the input admittance. The input admittance is then split into two parts: the lowpass 2-port and the highpass 2-port. Synthesis is then carried out on the two separate 2-ports. A fundamental theory in network synthesis describes the necessary and sufficient conditions for network realizability:

Theorem 2.1 (Darlington's Theory (19)): A given rational function Z(s) (Y(s)) is realizable as the driving-point impedance (admittance) of a passive lumped lossless reciprocal 2-port terminated in a resistor if and only if Z(s) (Y(s)) is a positive real function.

Definition 2.1 (Positive real function): A rational function, F(s), of the complex frequency $s = \sigma + j\omega$ is positive real if and only if the following conditions are satisfied:

(i) F(s) is real when s is real.

(ii) F(s) has no poles in the open RHP.

(iii) Poles of F(s) on the $j\omega$ axis, if they exist, are simple, and residues evaluated at these poles are real and positive.

(iv) Re $\{F(j\omega)\} \geq 0$ for all ω.

Once the 2-port input admittance is known, and if it is positive real, two methods are applicable for the synthesis. The first method, Darlington's method (3,19), uses an elegant relationship between the impedance and admittance parameters (z- and y-parameters) of the network, and the odd and even parts of the input impedance of the network. With these relationships, the realization is carried out by direct substitution. The second method is the cascade method (2,12). This method realizes the complete 2-port by successive reductions of the input impedance into a subnetwork with a positive real remainder. This process is continued until the input impedance function has been exhausted. In the general case, transformers may be required.

If one of the transmission coefficients of the 3-port network is used as a starting point, then relationships between the transmission coefficients and the voltage transfer ratios of the separate 2-ports must be developed. This is done taking into account the specific structure of the 3-port network. Once this has been accomplished, then well-known relationships are used to express the transfer function in terms of the y-parameters. Then Cauer ladder synthesis (18) may be applied to realize the 2-ports individually. This method is similar to the cascade method, in that it successively reduces the driving point impedance of the 2-port to a constant, except that it works to realize the zeros of z_{21} and z_{22} simultaneously. This results in slightly different logical processes between the two methods. The synthesis may require a variation of the Cauer ladder synthesis, in which parallel ladders are synthesized to realize complex transmission zeros in the 2-ports.

An alternative to the Cauer ladder synthesis can be used after the voltage transfer ratios for the two 2-ports are known. The methods of Bode or Brune and Gewertz (18) can be used to transform the transfer impedances Z_{21} and Z_{31}, which are easily derived from the voltage transfer ratios, into the input impedance for the two 2-ports. Then, Darlington synthesis can be carried out to realize the 2-ports.

2. Transformerless realizations

A concern, although not a central issue, to this research was to find synthesis methods which resulted in networks which do not use transformers. Transformers are expensive, difficult to manufacture to tight tolerances, and bulky compared to modern circuit elements. It is therefore desirable to find ways to avoid their use in modern circuits.

III. LITERATURE REVIEW

The research in the area of filter pairs and filter groups is, fortunately, very well documented and easy to follow. Most of the research was done during the 1960's and later, and as a result, is documented in computer databases such as Compandex and NTIS. The search for prior art in this area of 3-port synthesis was done by an electronic search through several databases using appropriate keywords. The electronic search yielded the paper which suggested this research, as well as several other important papers in this area. More importantly, the search yielded no citations which suggested that the research in this dissertation has already been done or is in progress by someone. Following are sections which document the research to be done in this area and the base for this study.

A. Norton's Constant Resistance Filter Pairs

The use of constant resistance networks was widespread in the American Telephone system when Norton wrote his famous paper. Otto Zobel (27) had written a paper several years earlier which defined the use of constant resistance filters for use in the telephone system to adjust signals for phase delays and distortion caused by long transmission paths. The concept of groups and supergroups, both terms referring to multiplexing many telephone signals onto one transmission line, was being discussed, and Norton concluded that the constant resistance filters were the best building block to use in the system. In the telephone system at that time, the filters were always used singly, and Norton set out to find ways of connecting the filters in parallel so that the networks could be used in the system of groups and supergroups.

Norton pointed out in his paper that the constant resistance filter pairs exhibited a necessary 3 dB insertion loss at the crossover frequency. His solution to this problem was to use more stages of the filter pairs.

B. Bennett's Filter Pairs

Bennett (11) investigated the effect of connecting the prototype 2-port lowpass and highpass filters in parallel, without regard to the loading effects each filter presents to the other. His results are valuable in that they point out that in some cases, the basic characteristics of either 2-port filter in the pair are lost due to the mutual loading effect of the filters when they are connected in parallel. His most important result is that when the corner frequencies of the prototype lowpass and highpass filters are very disparate,

$f_l \ll f_h$, the two filters behave as if they are not connected together. In this case, the input impedance is not constant, and the filter pair is a case covered in Belevitch's later paper.

C. Belevitch's Filter Pairs

Vitold Belevitch gathered all the articles established after World War II and summarized their contents in a 1958 paper (4). This paper is very interesting because it ties together the results from many researchers who were operating independently, and ultimately discovered the same principles. Image parameter design theory, for example, was the dominant method used for filter design in the years before World War II, and insertion-loss theory development started following the war. Belevitch points out that the developments of both were becoming very similar, and that a unified theory of filter design could be close at hand. The work done by Norton, Bennett, Zobel, Cauer, and many other researchers is discussed. Germane to this research is the fact that Norton's work was critiqued to some extent, and this summary adds a sense of history to the work done for this dissertation.

Carlin (13) wrote the initial paper which summarized the use of the scattering parameters in network analysis and synthesis. Belevitch (5) worked at the same time to define how the scattering parameters could be used to simplify the derivation of the network theories, and how the scattering parameters predicted results previously unknown in network theory. Carlin carried on to write texts on circuit theory, and Belevitch (6) carried on to explore the use of the scattering parameters in network synthesis. The primary focus of his work was in using the scattering parameters to predetermine the responses of the networks.

Belevitch saw a need for a filter pair which was at once canonical, having as few components as possible for the desired results, and having a greater insertion loss than 3 dB at the crossover frequency. Several of his previous papers had dealt with the development of the theory of scattering parameters of networks (7). He then wrote several papers dealing with the application of the scattering parameter design methods to filter pairs (8). These papers are important to this research, because in them Belevitch develops the relationships of the scattering parameters to the 3-port filters. Further developments discussed the synthesis of one-port filters, which can be treated as 3-port filters with resistive terminations.

D. Belevitch's Butterworth Filter Pairs

After developing the theory underlying the use of the scattering parameters as a starting point for the synthesis of 3-port filters, Belevitch (9) developed the theory of 3-port filters which have Butterworth filter pairs, which are filter pairs consisting of a lowpass filter in parallel with a highpass filter, each filter having the maximally-flat attenuation characteristic in its passband. Developments were shown for the cases in which the filters are complementary pairs, and in which the filters are of unequal orders. The Butterworth filter pairs were shown to be automatically complementary when the lowpass and highpass filters are of equal order.

E. Chen and Zhu's Butterworth Diplexers

Filter pairs are often called diplexers. Chen and Zhu (26), independently of Belevitch, developed a method for designing Butterworth filter pairs. Their development was quite similar to Belevitch's, except that Belevitch's derivation was more algebraic, and Chen's was more numerical. Both methods assumed that the networks were constructed of ladder filters connected in parallel.

F. Literature on Network Synthesis

Half of the work for this dissertation consisted of finding a straightforward method for synthesizing the Chebyshev and elliptic filter pairs once the scattering parameter description was known. Several texts were consulted in the effort to do this. Early texts which describe the Darlington and cascade syntheses in detail are Van Valkenburg (25), Storer (24), Karni (22) and Balabanian (3). Newer texts which give a matter-of-fact presentation of these methods are Chen (18) and Baher (2). The derivations of the methods are not quite so rigorous in these later texts, and the lack of distracting detail makes the methods more apparent.

G. Literature on Scattering Parameters

The concept of scattering parameters used to completely characterize a network is very powerful. Many aspects of a network's behavior can be readily seen by inspection of the scattering matrix for the network. Conversely, with knowledge of the relationship of the scattering matrix to the network properties, network behavior can be prescribed by the proper construction of the scattering matrix. Belevitch used this technique in his derivation of the Butterworth 3-ports. He deduced the form of the reflection coefficient scattering parameter, and used the form to construct the network.

Texts which describe the derivation and use of the scattering parameters in network theory are Chen (17), Carlin and Giordano (14), and Chan (16). Belevitch's (10) text is also a good reference.

IV. METHODS AND RESULTS

A. Scattering Theory of 3-port Networks

Central to the development of the Chebyshev and elliptic 3-port filters is the understanding of the scattering parameters. The reader is directed to the texts cited in the literature review for a complete background in the development of the scattering theory. This section will deal only with the aspects of the scattering theory which are germane to the development of the 3-port networks. The discussion will start with the 1-port scattering parameters and will then extend the essential theory to 3-port networks.

1. 1-port scattering parameters

The foundation for the scattering parameters lies in the understanding that an electrical wave, like any harmonic wave, can be studied as if it were composed of an incident part and a reflected part. The development of the scattering parameters starts with the definition of the port voltages and currents as incident and reflected components of the total voltage and current at the ports. This is illustrated in figure 4.1. The voltage and current are assigned subscripts to denote the fact that they are either incident or reflected waves. In the real world, the terms incident and reflected are meant to indicate the direction of travel for currents: currents either enter or exit a port. A similar interpretation exists for voltages: voltages are either driven into a port or they are impressed at the port by voltages at other ports.

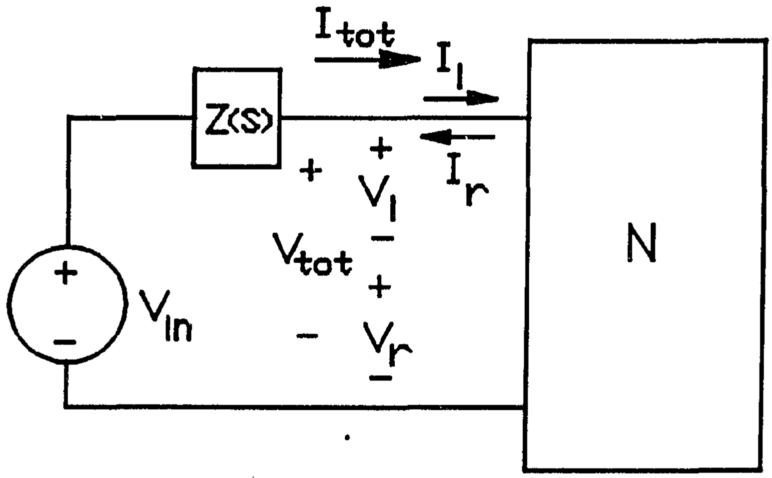

Figure 4.1. Example of a simple 1-port network showing the definitions for the incident and reflected currents and voltages

The total voltage and current at a port is the sum of the incident and reflected voltages and currents:

$$V_i + V_r = V_{tot}$$
$$I_i - I_r = I_{tot} \qquad (4.1)$$

The scattering parameters for a 1-port are identified as the reflection coefficient and the transmission coefficient. The scattering theory basically states that, at any given instant, the power entering (incident upon) and leaving (reflected from) the network must equal the total power supplied to the network. This is also stated: All of the power supplied to a network must be either reflected from the network or transmitted into the network. With the voltage and current definitions given in figure 4.1, the reflection coefficient for the 1-port is given as:

$$\rho(s) = \frac{V_{tot}(s) - I_{tot}(s)Z(s)}{V_{tot}(s) + I_{tot}(s)Z(s)} \qquad (4.2)$$

The transmission coefficient, $\tau(s)$, is related to the reflection coefficient by the power-conservation property stated earlier:

$$\rho^2(s) + \tau^2(s) = 1 \qquad (4.3)$$

or

$$\tau^2(s) = 1 - \rho^2(s) \tag{4.4}$$

This relationship is the main defining relationship of interest to this research. Other considerations for the scattering parameters arise from the power distribution through the network. Since the network is passive, and can therefore add no power to the power supplied by the source, the reflection and transmission coefficients are in themselves bounded by unity:

$$|\rho(s)| \leq 1 \tag{4.5}$$

and

$$|\tau(s)| \leq 1 \tag{4.6}$$

When the reflection and transmission coefficients for a network are constructed, this must be taken into consideration, and forms must be found which ensure boundedness.

2. 3-port extensions

The essence of the scattering parameters does not change when the network grows from one to three ports. Each port has a pair of transmission coefficients which describe how power flows from that port to the other two ports. Each port also has a reflection coefficient which describes how power is reflected back to the load at that port. Figure 4.2 shows a schematic representation of a general 3-port filter with a voltage source at port 1, and load resistances at ports 2 and 3. The figure also defines the incident and reflected waves a(s) and b(s), respectively. These waves are used in the definition of the port transmission and reflection coefficients. The waves are in units of the square root of power.

When the network is expanded from a 1-port to a 3-port, the scattering parameters are expressed in matrix form. The matrix is n x n, where n is the number of ports in the network, three in this case. The 3 x 3 matrix is designated S(s), and is written as:

$$S(s) = \begin{bmatrix} S_{11}(s) & S_{12}(s) & S_{13}(s) \\ S_{21}(s) & S_{22}(s) & S_{23}(s) \\ S_{31}(s) & S_{32}(s) & S_{33}(s) \end{bmatrix} \tag{4.7}$$

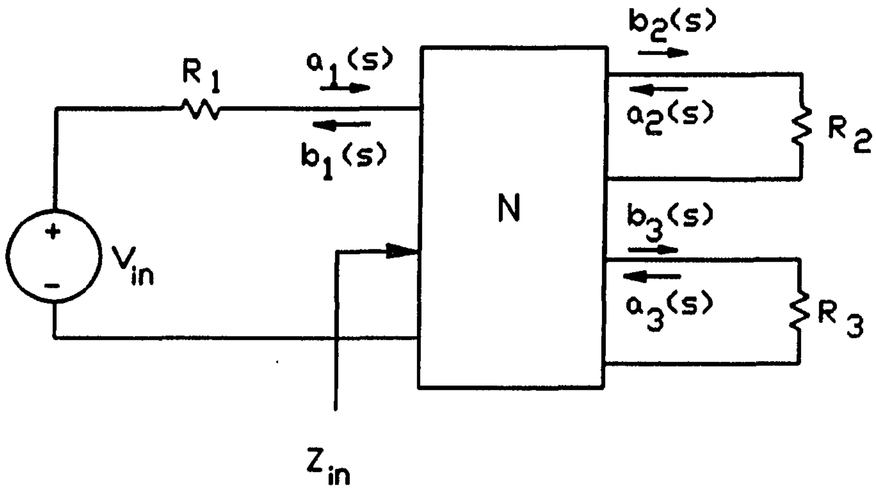

Figure 4.2. Example of a simple 3-port showing the definitions of the incident and reflected waves at the ports

The main diagonal elements in the matrix are the reflection coefficients, which are defined as follows:

$$S_{jj}(s) = \left.\frac{b_j(s)}{a_j(s)}\right|_{a_x(s)=0 \text{ for } x \neq j} \tag{4.8}$$

This definition states that the reflection coefficient at any port is equal to the ratio of the Laplace transform of the reflected wave to the Laplace transform of the incident wave, given that the other two ports are undriven, and are terminated in their load impedances.

The off-diagonal elements of the matrix are the transmission coefficients for the 3-port and are defined as follows:

$$S_{mj}(s) = \left.\frac{b_m(s)}{a_j(s)}\right|_{a_x(s)=0 \text{ for } x\neq j} \tag{4.9}$$

This definition states that the transmission from one port (port m) to another (port j) is equal to the ratio of the Laplace transform of the reflected wave at the receiving port to the Laplace transform of the incident wave at the transmitting port, again assuming that the receiving ports are undriven and are terminated in their load impedances.

One condition that the scattering matrix of a lossless network must satisfy is that it must be bounded-real and paraunitary. The definitions for these properties are as follows:

DEFINITION 1.0: A square matrix A(s) is said to be bounded-real if it satisfies the following conditions:

(i) $\bar{A}(s) = A(\bar{s})$ for all s in the open RHP

(ii) each of the elements of A(s) is analytic in the open RHP (4.10)

(iii) $1 - \sum_{j=1}^{n} |S_{ji}(j\omega)|^2 \geq 0$ (i = 1 to n) for all ω

where $\bar{A}(s)$ is defined as the matrix of functions created by negating the odd parts of the functions in the matrix A(s), and $A(\bar{s})$ are the elements of A(s) with the complex conjugate of s substituted for s ($\bar{s}$ is the complex conjugate of s).

The first condition states that the elements of S(s) are real if s is real. This can be ensured if the numerator and denominator polynomials of the elements have real coefficients. The second requirement states that the elements of S(s) can have no poles in the open right half of the complex s-plane (RHP). The third requirement states that each of the scattering parameters must be bounded by unity. Since the network is passive, no power can be added to that supplied by the source, so therefore the transmission and reflection coefficients cannot be greater than one.

DEFINITION 2.0: An n x n matrix A(s) is called paraunitary if

$$A(s)A(-s) = U_n, \tag{4.11}$$

where U_n is the nth-order unit matrix.

The paraunitary n x n matrix A(s) yields six equations which define the magnitude and phase relationships between the scattering parameters:

$$S_{11}(s)S_{11}(-s) + S_{12}(s)S_{12}(-s) + S_{13}(s)S_{13}(-s) = 1 \tag{4.12a}$$
$$S_{12}(s)S_{12}(-s) + S_{22}(s)S_{22}(-s) + S_{23}(s)S_{23}(-s) = 1 \tag{4.12b}$$
$$S_{13}(s)S_{13}(-s) + S_{23}(s)S_{23}(-s) + S_{33}(s)S_{33}(-s) = 1 \tag{4.12c}$$
$$S_{11}(s)S_{12}(-s) + S_{12}(s)S_{22}(-s) + S_{13}(s)S_{23}(-s) = 0 \tag{4.12d}$$
$$S_{11}(s)S_{13}(-s) + S_{12}(s)S_{23}(-s) + S_{12}(s)S_{33}(-s) = 0 \tag{4.12e}$$
$$S_{12}(s)S_{13}(-s) + S_{22}(s)S_{23}(-s) + S_{23}(s)S_{33}(-s) = 0 \tag{4.12f}$$

Equation (4.12a) will be used in the derivation of the transmission and reflection coefficients for the 3-port networks.

DEFINITION 3.0: An n x n rational matrix is the scattering matrix of a linear, lumped, time-invariant and lossless n-port network, normalizing to the n load resistances at the ports, if and only if it is bounded-real and paraunitary.

The rules for the scattering matrix of the 3-ports are now specified, and with them are the conditions for the individual scattering parameters. The scattering parameters necessary to characterize and synthesize the Chebyshev and elliptic 3-port networks can now be theorized.

3. Development of the scattering parameters

This section details the development of the general form of the scattering parameters for the filter pairs. The logic of the development follows Belevitch (9).

In all of the following sections, the frequency variable s will be restricted to the real-frequency axis, represented by $s = j\omega$. For convenience, some of the equations will be expressed as functions of s, and where clarity is of utmost importance, equations will be expressed as functions of $j\omega$.

Consider the transmission of power from port 1 to port 2, the lowpass filter pathway. This is characterized by the transmission coefficient s_{12}. Define a Zero of Attenuation (attenuation zero) as the case in which all of the power input at port 1 is being transmitted to one of the other two ports. At frequencies at which the lowpass attenuation zeros occur,

$$1 - S_{12}(j\omega)S_{12}(-j\omega) = 0 \qquad (4.13)$$

(4.12a) can be rearranged:

$$1 - S_{12}(j\omega)S_{12}(-j\omega) = S_{11}(j\omega)S_{11}(-j\omega) + S_{13}(j\omega)S_{13}(-j\omega) \quad (4.14)$$

At an attenuation zero in the lowpass direction, the left side of (4.14) is zero. Since $S_{11}(j\omega)S_{11}(-j\omega)$ and $S_{13}(j\omega)S_{13}(-j\omega)$ are squares of magnitudes, and are therefore positive for all frequencies, the right side of (4.14) can be zero only if $S_{11}(j\omega)S_{11}(-j\omega) = 0$ and $S_{13}(j\omega)S_{13}(-j\omega) = 0$. Similarly, for the highpass direction, $1 - S_{13}(j\omega)S_{13}(-j\omega) = 0$ only when $S_{11}(j\omega)S_{11}(-j\omega) = 0$ and $S_{12}(j\omega)S_{12}(-j\omega) = 0$. Thus, the attenuation zero in the lowpass filter is accompanied by transmission zeros in the highpass direction, and in the reflection direction. The Chebyshev and elliptic filters are characterized by having ripples in the passband, and a steeper attenuation characteristic than the Butterworth filters. The ripples are of constant amplitude and have a peak value of unity. The frequencies at which the ripple peaks occur are the frequencies of the attenuation zeros, and are the frequencies at which $F_n(s) = 0$.

The transducer power gain for the lowpass Chebyshev or elliptic 2-port filter is given by:

$$S_{12}(s)S_{12}(-s) = \frac{1}{1+\epsilon^2 F_n^2(s)} \qquad (4.15)$$

where ϵ is the ripple factor

$F_n(s)$ is an approximation function (Chebyshev or elliptic)

and the corresponding expression for the highpass filter is given by:

$$S_{13}(s)S_{13}(-s) = \frac{1}{1+\epsilon^2 F_n^2(1/s)} \qquad (4.16)$$

where ϵ is the ripple factor

$F_n(1/s)$ is an approximation function (Chebyshev or elliptic)

Thus a logical starting point in the construction of the transmission coefficients for the 3-port is to develop a form of (4.12a) which is based on (4.15) and (4.16). Each of the three factors in (4.12a) can be rational, and each must have the same denominator. It is known from (4.14) and the previous discussion that when either of the transmission pathways has an attenuation zero, the magnitude of the reflection coefficient, as well as the other transmission factor, must have transmission zeros. This is true for attenuation zeros in the lowpass direction as well as the highpass direction. Therefore, the term involving the reflection coefficient, $S_{11}(j\omega)S_{11}(-j\omega)$, must have zeros for all of these attenuation zeros. Since the attenuation zeros of the transmission coefficients are the frequencies at which the Chebyshev or elliptic polynomial have zeros, the polynomial itself can be used as the numerator term in the factor. The form of the numerator of the reflection coefficient term, $S_{11}(s)S_{11}(-s)$, can therefore be:

$$F_n^2(s)F_n^2(1/s) \tag{4.17}$$

The two transducer power gains should exhibit transmission zeros when their counterparts exhibit attenuation zeros. In order to ensure this, the numerators of the lowpass and highpass transmission terms can be made to contain the approximation polynomial from the denominator of the highpass and lowpass transmission terms, respectively. Thus, as a polynomial zero induces an attenuation zero in one transmission term, it provides a transmission zero for the other transmission term. The common denominator for the three terms can be the product of the denominators from (4.15) and (4.16):

$$\frac{1}{[1 + \epsilon^2 F_n^2(s)][1 + \epsilon^2 F_n^2(1/s)]} \tag{4.18}$$

In order to preserve the form of (4.15) and (4.16) while including the common denominator in (4.18), the numerators of

(4.15) and (4.16) are multiplied by the term in the denominator of (4.18) which is missing in the denominator of (4.15) and (4,16). The scattering parameter equation theorized up to this point can be written as:

$$\frac{F_n^2(s)\,F_n^2(1/s)}{[1 + \epsilon^2 F_n^2(s)][1 + \epsilon^2 F_n^2(1/s)]} + \qquad (4.19)$$

$$\frac{1 + \epsilon^2 F_n^2(1/s)}{[1 + \epsilon^2 F_n^2(s)][1 + \epsilon^2 F_n^2(1/s)]} +$$

$$\frac{1 + \epsilon^2 F_n^2(s)}{[1 + \epsilon^2 F_n^2(s)][1 + \epsilon^2 F_n^2(1/s)]} = 1$$

This equation does not balance, but can be made to balance if the "1" terms in the numerators of the second and third terms are made to equal 0.5, and the numerator of the first term is multiplied by ϵ^4. The result is:

$$\frac{\epsilon^4 F_n^2(s)\,F_n^2(1/s)}{[1 + \epsilon^2 F_n^2(s)][1 + \epsilon^2 F_n^2(1/s)]} + \qquad (4.20)$$

$$\frac{0.5 + \epsilon^2 F_n^2(1/s)}{[1 + \epsilon^2 F_n^2(s)][1 + \epsilon^2 F_n^2(1/s)]} +$$

$$\frac{0.5 + \epsilon^2 F_n^2(s)}{[1 + \epsilon^2 F_n^2(s)][1 + \epsilon^2 F_n^2(1/s)]} = 1$$

The choice of 0.5 as the constants in tne second and third terms of (4.20) is done to preserve to symmetry of the magnitude responses. Other combinations of constants are possible, but were not investigated. The sum of the constants must be 1.

The squared magnitudes of the scattering parameters are thus:

$$S_{11}(s)S_{11}(-s) = \frac{\epsilon^4 F_n^2(s) F_n^2(1/s)}{[1 + \epsilon^2 F_n^2(s)][1 + \epsilon^2 F_n^2(1/s)]} \qquad (4.21a)$$

$$S_{12}(s)S_{12}(-s) = \frac{0.5 + \epsilon^2 F_n^2(1/s)}{[1 + \epsilon^2 F_n^2(s)][1 + \epsilon^2 F_n^2(1/s)]} \qquad (4.21b)$$

$$S_{13}(s)S_{13}(-s) = \frac{0.5 + \epsilon^2 F_n^2(s)}{[1 + \epsilon^2 F_n^2(s)][1 + \epsilon^2 F_n^2(1/s)]} \qquad (4.21c)$$

4. Perfect Chebyshev and elliptic filter pairs

A perfect Chebyshev or elliptic filter pair can be described as having pure Chebyshev or elliptic transmission characteristics in the lowpass and highpass directions. In practice, this implies that the transducer power gains $S_{12}(s)S_{12}(-s)$ and $S_{13}(s)S_{13}(-s)$ are of exactly the forms described by (4.15) and (4.16), respectively. Using this ideal form for the transducer power gains, (4.12a) becomes

$$\frac{F_n^2(s)F_n^2(1/s)-1}{[1+\epsilon^2F_n^2(s)][1+\epsilon^2F_n^2(1/s)]} + \qquad (4.22)$$

$$\frac{1+\epsilon^2F_n^2(1/s)}{[1+\epsilon^2F_n^2(1/s)]}\,\frac{1}{[1+\epsilon^2F_n^2(s)]} +$$

$$\frac{1+\epsilon^2F_n^2(s)}{[1+\epsilon^2F_n^2(s)]}\,\frac{1}{[1+\epsilon^2F_n^2(1/s)]} = 1$$

The numerator of the first term, the reflection term, will be equal to -1 whenever the approximation polynomial F(s) is equal to zero. In the Chebyshev and elliptic cases, these zeros of F(s) are the frequencies at which the attenuation zeros occur, and they occur at finite frequencies in the passband, as opposed to the zero and infinite frequencies. Since the terms in (4.22) are all magnitudes, and therefore must always be positive, this negative value is not allowed. This argument shows that perfect Chebyshev or elliptic filter pairs do not exist.

Perfect Butterworth filter pairs do exist, however. If the approximation polynomial in (4.20) is that of the Butterworth characteristic, $F(s) = s^n$, then F(s)*F(1/s) = 1, and the numerator of the first term in (4.20) is equal to zero. The reflection coefficient is therefore zero. The resulting transducer power gains are those of the constant-resistance Butterworth filter pair.

5. Approximate Chebyshev and elliptic filter pairs

The transmission terms (4.21b) and (4.21c) each contain the same form as described in (4.15) and (4.16). However, each term also has a multiplicative term which is used to make the equation balance and match the form of (4.12a). These terms are

$$\frac{0.5 + \epsilon^2 F_n^2(1/s)}{1 + \epsilon^2 F_n^2(1/s)} \qquad (4.23)$$

in (4.21b) and

$$\frac{0.5 + \epsilon^2 F_n^2(s)}{1 + \epsilon^2 F_n^2(s)} \qquad (4.24)$$

in (4.21c). Equation (4.19) can be made to match the form of (4.12a) by subtracting unity in the numerator of the first term, the reflection term, of (4.19). This is not allowed, however, as pointed out in the last section. The multiplicative factors in (4.21b) and (4.21c) are necessary, then, and represent a departure from the ideal desired result. The result is that the transducer power gains and reflection coefficients described in (4.21) are approximations to those of the perfect filter pairs. The question which arises is "How well does the approximation represent the ideal case?"

The traditional reason for using the Chebyshev or elliptic filters is to get a faster attenuation characteristic beyond the cutoff frequency. That criterion will be used here to determine whether the approximation is good enough to be used.

These functions were simulated using the MATHCAD personal computer utility. Fourth-order Chebyshev polynomials were used as the approximation polynomials. Plots of the transducer power gains (4.21b) and (4.21c) vs. frequency for the lowpass and highpass filters are shown in figure 4.3. Figure 4.4 shows the desired transducer power gain response from a perfect Chebyshev pair. Figure 4.5 shows a plot of (4.23), the factor associated with the lowpass filter, and also a plot of the perfect Chebyshev lowpass filter transducer power gain response, based on (4.15). The resulting lowpass response in figure 4.3 results from the product of the two plots in figure 4.5. The multiplicative factor can be seen to decrease in value before the Chebyshev response does, and this causes the product of the two to decrease also, causing an effective rounding off of the Chebyshev response, as seen in figure 4.3, The multiplicative factor decreases to a limit value of 0.5, and the effect of this is to cause the response in figure 4.3 to decrease faster than the Chebyshev response, thereby preserving the faster attenuation characteristic of the Chebyshev filter.

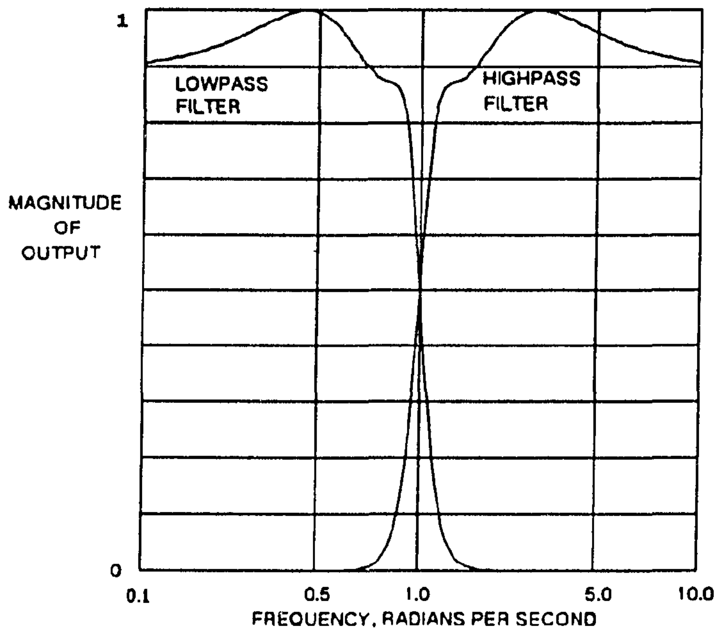

Figure 4.3. Plot of transducer power gain vs. frequency for derived Chebyshev filter pair

A comparison of figures 4.3 and 4.4 reveals the effect of the factors (4.23) and (4.24). These factors cause the behavior of the individual filters to alter around the cutoff frequency. The peak in the response near the corner frequency in figure 4.4, the ideal Chebyshev pair, is missing and rounded off in the proposed filter pair, figure 4.3. The

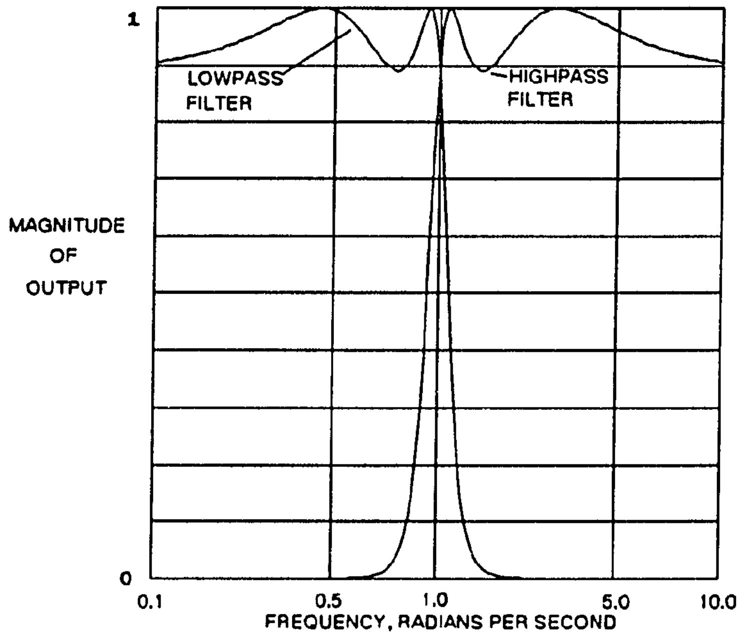

Figure 4.4. Plot of transducer power gain vs. frequency for perfect Chebyshev filter pair

slope of the response characteristic in the transition region remains steeper than in the Butterworth filter pairs, however. This is the desirable characteristic for the Chebyshev filters. The transducer power gain and the reflection coefficients derived in (4.21) are therefore valid for use as approximations to the Chebyshev filter pairs. Similar results were obtained for the third-order elliptic filter pairs.

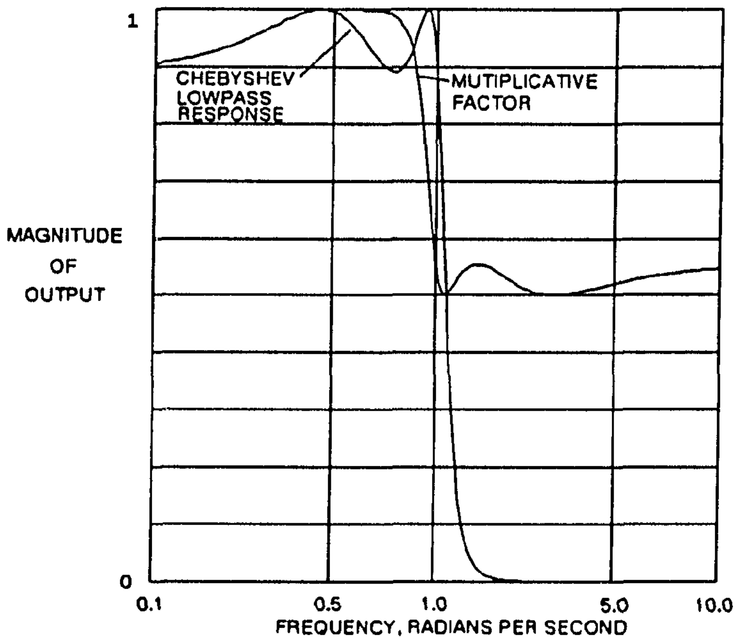

Figure 4.5. Plot of perfect Chebyshev transducer power gain and the multiplicative error factor which appears in the proposed filter pair

6. Positive-real input admittance

In order for the filter pair to be physically realizable, its input admittance must be positive real, and the input admittance of the associated lowpass and highpass filters must also be positive real. The input admittance for the network as calculated with (2.1) is always positive real, as demonstrated by the following proof:

Theorem 4.1 (Positive real admittance): The input admittance, Y_{in}, of the lumped passive reciprocal 3-port as calculated with (2.1) using (4.5) is always positive real.

Proof: Necessary and sufficient conditions for a complex-valued rational function, $Y_{in}(s)$, of the complex frequency, s, to be positive real are:

a) $Y_{in}(s)$ is real when s is real, and

b) $Re(Y_{in}(s)) \geq 0$ when $Re(s) \geq 0$.

Since $\rho(s)$ is a rational function with real coefficients, its substitution into Y_{in} will not yield an input impedance with complex coefficients. Condition a) is therefore satisfied by (2.1). Condition b) can be shown to be satisfied by expanding the real part of (2.1) with ρ expanded as a+jb, where j is the imaginary unit, as follows:

$$Re\,(Y_{in}) = \frac{1-(a^2+b^2)}{1+2a+(a^2+b^2)} \qquad (4.25)$$

The numerator of (4.25) is positive if the modulus of a and b, $a^2 + b^2$, is less than 1. Since the magnitude of the reflection coefficient is the modulus of a and b, and is always less than 1, then the numerator of (4.25) will always be positive, and condition b) is satisfied. Therefore, Y_{in} is always positive real.

B. General Method

In the following sections, the scattering parameters and networks will be developed for the third- and fourth-order cases of the Chebyshev and elliptic networks. A general method, or algorithm, will be used. This algorithm consists of the following steps:

1. Choose an appropriate approximating function. A Chebyshev polynomial of the first kind is appropriate for the Chebyshev filter, and a Chebyshev rational function resulting from the Jacobian elliptic integral of the first kind is appropriate for the elliptic filters.

2. Calculate an expression for the magnitude-squared value of the reflection coefficient, using (4.21a) with the approximation function.

3. Factor the magnitude-squared of the reflection coefficient into its left and right half of the s-plane (LHP and RHP) poles and zeros.

4. Form the minimum-phase reflection coefficient from the LHP poles and zeros found in step 3. Use of the minimum-phase reflection coefficient maximizes the DC gain of the lowpass filter in the pair, and the gain at infinity for the highpass filter (17).

5. Calculate Y_{in} from (2.13) based on the reflection coefficient found in step 4. This admittance will be positive-real.

6. Expand Y_{in} found in step 5 into two separate parts conforming to the input impedance of standard lowpass and highpass filters:

$$Y_{lowpass} = \frac{a_{n-1}s^{n-1} + \dots + a_0}{b_n s^n + \dots + b_0} \tag{4.26}$$

$$Y_{highpass} = \frac{s(c_n s^{n-1} + \dots + c_1)}{d_n s^n + \dots + d_0} \tag{4.27}$$

7. Test the input impedances for the two 2-ports to be sure that they are positive real. If they are not, the synthesis is not possible.

8. Synthesize the input impedances using Darlington's method, assuming a 1-ohm load resistor.

Steps 6 and 7 require the positive-real input admittance to be expanded into two separate parts, one representing the input admittance of the lowpass filter and the other representing the input admittance of the highpass filter. In general, the input admittance of an nth-order filter pair will be a rational function numerator and denominator of degree 2n. The denominator of the admittance is comprised of the 2n distinct zeros found in step 3. In general, the even-order filter pairs will have n quadratic factors in the denominator of Y_{in}, and the odd-order filter pairs will have n quadratic factors and 2 factors on the negative real axis. It is not immediately obvious which quadratic factors and real factors are used together to form the denominators of the input admittances for the lowpass and highpass filters. Since there are 2n factors which need to be joined into two denominators, there are several combinations which must be tried in order to find combinations which yield positive-real input admittances for the lowpass and highpass filters. The number of combinations for the even-order networks is equal to 2n objects taken n at a time: 2 for the 2nd-order network, 6 for the 4th-order network and so forth. The number of combinations for the odd-order networks is two times that of the next lower order network: 4 for the 3rd-order network, 12 for the 5th-order network and so forth.

The criterion for acceptance of one expansion trial is that all of the coefficients a and b in (4.26) and (4.27) be strictly positive. Zero coefficients are not allowed, since positive-real rational functions must not have missing terms in the numerator and denominator polynomials (18). The trials which meet the criteria must still be tested for positive-realness, since it has yet to be proven that the expansion of a positive-real admittance always yields two positive-real admittances which conform to (4.26) and (4.27).

C. Chebyshev 3-port Filters

1. Second-order filter pairs

The approximation function chosen is the second order Chebyshev polynomial:

$$C_2=2\omega^2 - 1 = \cos(2 \cos^{-1} \omega) \qquad (4.28)$$

This function is used in (4.20a) to create the resulting reflection coefficient magnitude:

$$S_{11}(s)S_{11}(-s) = \frac{0.0595s^8+0.2980s^6+0.4910s^4+0.2980s^2+0.0595}{0.548s^8+0.786s^6+1.735s^4+0.786s^2+0.548} \quad (4.29)$$

Since

$$|S_{11}(\omega)|^2 = S_{11}(j\omega)S_{11}(-j\omega) \quad (4.30)$$

the reflection coefficient $S_{11}(\omega)$ can be created from the LHP roots of its magnitude-squared. The roots of (4.29) are found using a computer program which implements Laguerre's method to find the complex roots of a complex polynomial. The roots are listed in table (4.1).

The reflection coefficient, $S_{11,}$ is created from the LHP roots listed in table 4.1. as follows: The reflection coefficient, as previously mentioned, must be bounded real, and can therefore have no RHP poles. Therefore, the denominator of the reflection coefficient is constructed using the LHP roots in table 4.1. The reflection coefficient is desired to be minimum phase, having no RHP zeros, so the LHP numerator roots in table 4.1 are used to create the numerator. The result is

$$S_{11}(s) = \frac{0.244s^4+0.610s^2+0.244}{0.740s^4+1.750s^3+2.600s^2+1.750s^+0.740} \qquad (4.31)$$

The input admittance of the 3-port can be found directly using (2.13), and is found to be:

$$Y_{in} = \frac{0.495s^4+1.750s^3+1.990s^2+1.750s+0.495}{0.983s^4+1.750s^3+3.210s^2+1.750s+0.983} \qquad (4.32)$$

The input admittance can be split into two parts, representing the admittance seen at the input of the lowpass filter and the highpass filter. To do this, expressions (4.26) and (4.27) are used which represent the form of the input admittance for a lowpass and a highpass filter, respectively. These are summed, and equated to (4.32) as follows:

$$g_0 + \frac{a_{n-1}s^{n-1} + \dots + a_0}{b_n s^n + \dots + b_0} + \frac{s(c_n s^{n-1} + \dots + c_1)}{d_n s^n + \dots + d_0} = \frac{0.495s^4+1.750s^3+1.990s^2+1.750s+0.495}{0.983s^4+1.750s^3+3.210s^2+1.750s+0.983} \qquad (4.33)$$

The unknown coefficients of the resulting equation are then solved, and the result is:

$$Y_{in} = \frac{0.374s + 0.108}{s^2 + 0.575s+0.477} + \frac{s(0.227s+0.784)}{s^2+1.205s+2.095} + 0.2761 \qquad (4.34)$$

Table 4.1. Roots of Equation (4.29)

Root Number	Numerator Root Value	Denominator Root Value
1	+j1.4140	+0.4701+j0.6622
2	+j1.4140	+0.4701-j0.6622
3	-j1.4140	-0.4701+j0.6622
4	-j1.4140	-0.4701-j0.6622
5	+j0.7070	+0.7128+j1.0040
6	+j0.7070	+0.7128-j1.0040
7	-j0.7070	-0.7128+j1.0040
8	-j0.7070	-0.7128-j1.0040

The 0.2761 Siemens (mho) conductance represents a resistor which must be placed across the input port in order to make the input admittance separate into lowpass filter and highpass filter parts. This element represents the minimum real part of the input admittance when $s = j\omega$. This shunt resistor and the source resistor can be replaced by a Thevenin equivalent circuit. This new input circuit consists of a new source resistor which is equal to the parallel combination of the original resistors, and a new voltage source which is equal to

the original source times the voltage divider ratio of the original resistors. This replacement was avoided in this research because it was thought to be more important to show the actual derived structure.

The input admittances, and therefore the impedances, of the lowpass and highpass filters are now known, as well as the transmission characteristics of the overall lowpass and highpass pathways. The task now is to synthesize the lowpass and highpass filters as two 2-port networks as shown in figure 2.2. The Darlington procedure can be used to convert the input admittances of the two 2-ports into corresponding y_{22} and y_{21} parameters. Cauer synthesis can then be used to realize the filters (18).

The Darlington procedure along with the Cauer synthesis yields the network in figure 4.6. The circuit shown in figure 4.6 was simulated using the SPICE circuit analysis program. A plot of the frequency response of the circuit outputs is shown in figure 4.7. As can be seen, the frequency response at the passband frequency extreme (DC for the lowpass, infinity for the highpass) is attenuated by the ripple amount. This is one characteristic of Chebyshev filters. The response of the odd-order filters is not attenuated at the extremes. The passband of the two filters contains slightly more ripple than the 0.5 dB design value. The excess is not extreme, however. The

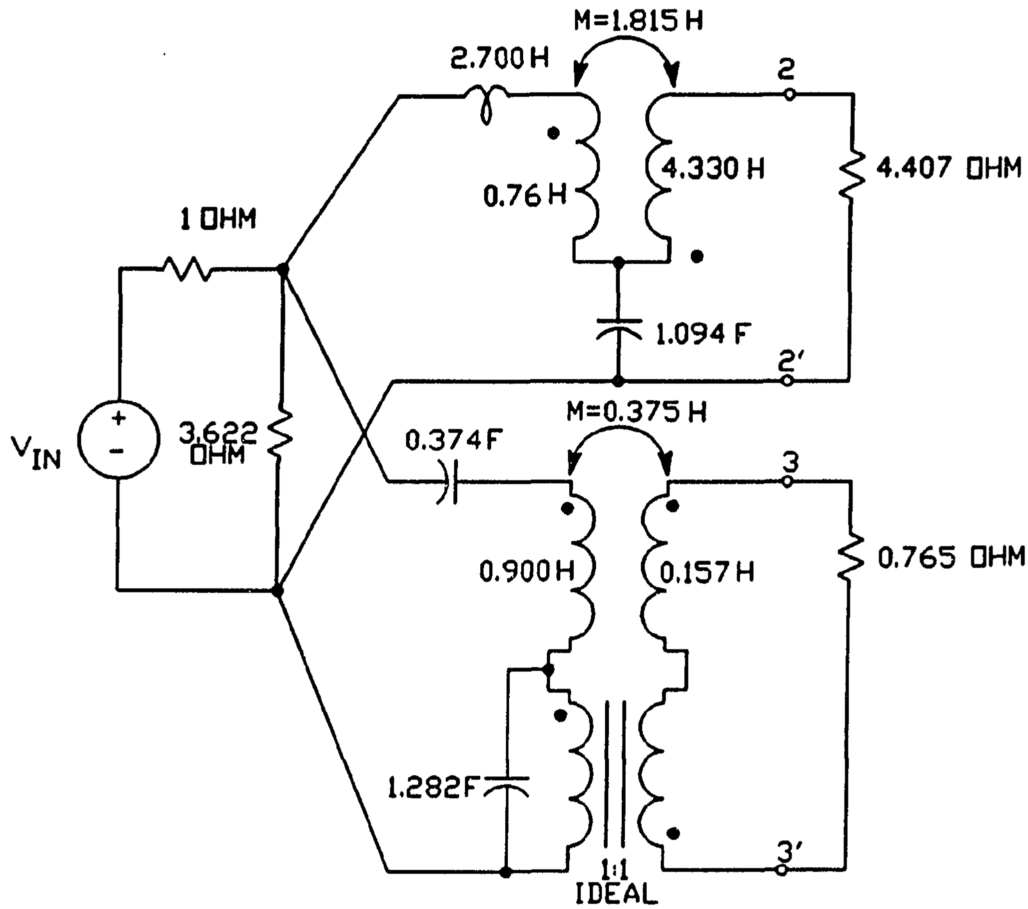

Figure 4.6. Second-order Chebyshev filter pair

highpass response is about 10 dB lower than the lowpass response. This effect is caused by the voltage ratio of the transformer which provides the pole at infinity, which in this case is the only transformer in the highpass filter.

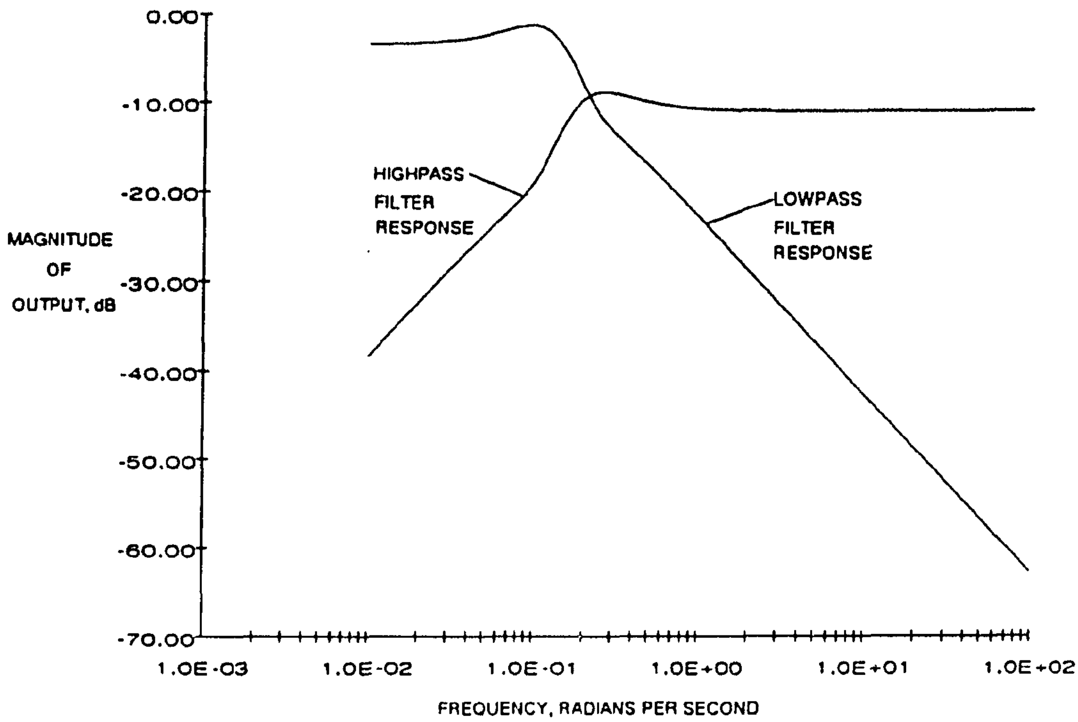

Figure 4.7. Frequency response of the second-order Chebyshev filter pair. Analysis was performed using the SPICE circuit analysis program

2. Third-order filter pairs

The approximation function chosen is the third-order Chebyshev polynomial:

$$C_3 = 4\omega^3 - 3\omega = \cos(3\cos^{-1}\omega), \qquad |\omega| \le 1$$

$$= \cosh(3\cosh^{-1}\omega), \quad |\omega| \ge 1 \qquad (4.35)$$

Use of this polynomial in (4.21a) results in the reflection coefficient magnitude:

$$S_{11}(s)S_{11}(-s) = \frac{-2.1433s^{10}-8.9304s^{8}-13.5891s^{6}}{1.9520s^{12}+0.7847s^{10}-7.8324s^{8}-14.5891s^{6}} \qquad (4.36)$$
$$\frac{-8.9304s^{4}-2.1433s^{2}}{-7.8324s^{4}+0.7847s^{2}+1.9520}$$

The roots of (4.36) are listed in table 4.2.

The minimum-phase reflection coefficient is created from the LHP roots of the numerator and denominator of (4.36) as listed in table (4.2). The result is:

$$S_{11}(s) = \frac{1.4640s^{5}+3.0500s^{3}}{1.3971s^{6}+4.7268s^{5}+8.3445s^{4}+10.6156s^{3}} \qquad (4.37)$$
$$\frac{+1.4640s}{+8.3445s^{2}+4.7468s+1.3971}$$

The input admittance is found directly from (2.13) and is:

Table 4.2. Roots of Equation (4.36)

Root Number	Numerator Root Value	Denominator Root Value
1	+j0.1547	+0.2742+j0.8945
2	+j0.1547	+0.2742-j0.8945
3	-j0.1547	-0.2742+j0.8945
4	-j0.1547	-0.2742-j0.8945
5	+j0.8660	+0.3132+j1.0219
6	+j0.8660	+0.3132-j1.0219
7	-j0.8660	-0.3132+j1.0219
8	-j0.8660	-0.3132-j1.0219
9	0	+0.6265
10	0	-0.6265
11		+1.5962
12		-1.5962

$$Y_{in} = \frac{1.3971s^6+3.2828s^5+8.3445s^4+7.5656s^3+8.3445s^2+3.2828s+1.3971}{1.3971s^6+6.2108s^5+8.3445s^4+13.6656s^3+8.3445s^2+6.2108s+1.3971} \qquad (4.38)$$

The input admittance is separated into lowpass and highpass filter parts as in the second-order case and the result is:

$$Y_{in} = \frac{0.4566s^2+0.1961s+0.1912}{s^3+0.6014s^2+0.8114s+0.2110} + \frac{0.9055s^3+0.9292s^2+2.1641s}{s^3+3.8440s^2+2.8493s+4.7378} + 0.9425 \tag{4.39}$$

The Darlington procedure with the Cauer synthesis yields the realization shown in figure 4.8.

The circuit shown in figure 4.8 was simulated using the SPICE circuit analysis program. The frequency response plots for the lowpass and highpass filters are shown in figure 4.9. The plots show the response at a maximum at the passband extremes for the two filters, which is expected for the odd-order Chebyshev filter. The passband ripple is about 0.5 dB, the design value.

3. Fourth-order filter pairs

The approximation function chosen is the fourth-order Chebyshev polynomial:

$$C_4(\omega) = 8\omega^4-8\omega^2+1 \tag{4.40}$$

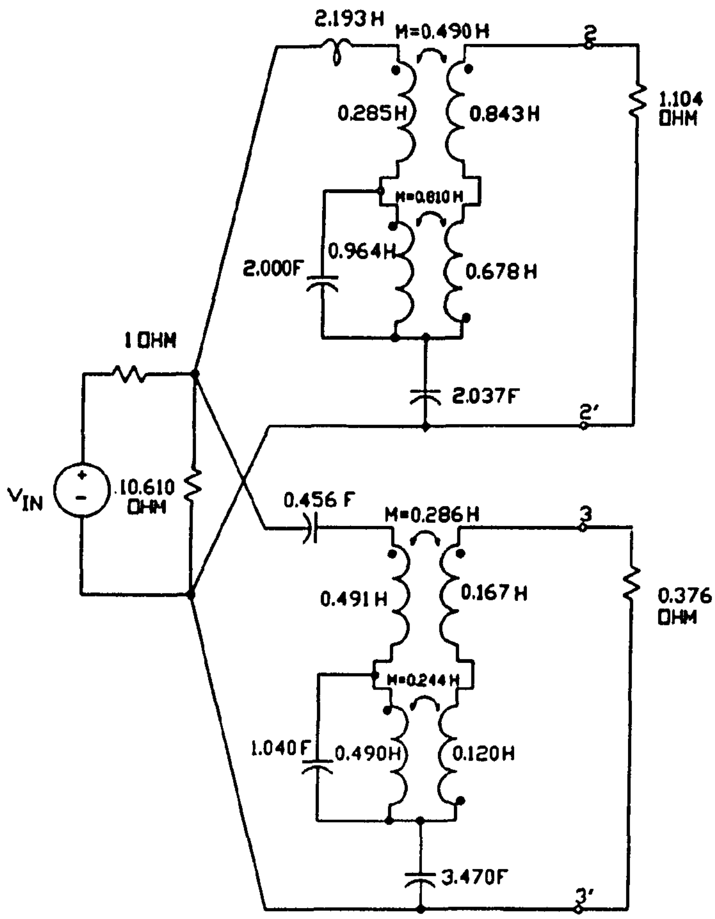

Figure 4.8. Third-order Chebyshev filter pair

The reflection coefficient magnitude is calculated using (4.21a) and is:

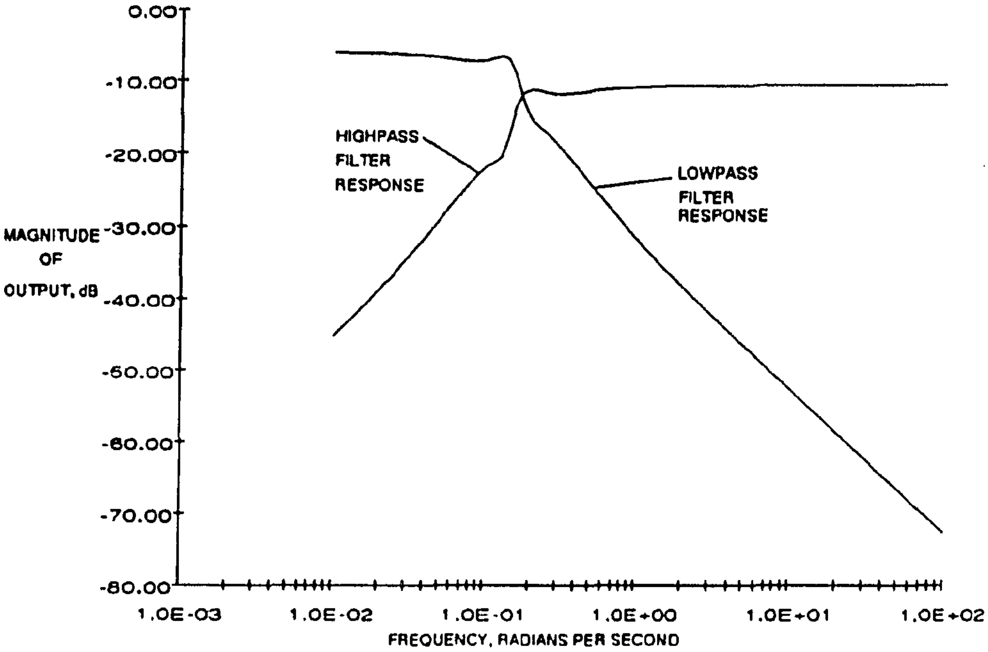

Figure 4.9 Frequency response of the third-order Chebyshev filter pair

$$S_{11}(s)S_{11}(-s) = \frac{.9526s^{16}+17.1463s^{14}+107.8790s^{12}}{8.7606s^{16}+32.7623s^{14}+117.6390s^{12}} \qquad (4.41)$$

$$\frac{+293.6310s^{10}+403.9060s^{8}+293.6310s^{6}}{+295.5830s^{10}+405.1510s^{8}+295.5830s^{6}}$$

$$\frac{+107.8790s^{4}+17.1463s^{2}+0.9526}{+117.6390s^{4}+32.7623s^{2}+8.7606}$$

Table 4.3. Roots of Equation (4.41)

Root Number	Numerator Root Value	Denominator Root Value
1	+j0.3827	+0.1649+j0.9555
2	+j0.3827	+0.1649-j0.9555
3	-j0.3827	-0.1649+j0.9555
4	-j0.3827	-0.1649-j0.9555
5	+j0.9239	+0.1754+j1.0163
6	+j0.9239	+0.1754-j1.0163
7	-j0.9239	-0.1754+j1.0163
8	-j0.9239	-0.1754-j1.0163
9	-j1.0824	+0.4234+j0.4209
10	-j1.0824	+0.4234-j0.4209
11	+j1.0824	+0.4234+j0.4209
12	+j1.0824	+0.4234-j0.4209
13	-j2.6131	+1.1878+j1.1810
14	-j2.6131	+1.1878-j1.1810
15	+j2.6131	-1.1878+j1.1810
16	+j2.6131	-1.1878-j1.1810

The roots of (4.41) are listed in table (4.3). The LHP roots in table (4.3) are used to construct the minimum-phase reflection coefficient:

$$S_{11}(s) = \frac{0.9760s^8+8.7839s^6+15.7375s^4}{2.9598s^8+11.5512s^7+28.0757s^6+42.1858s^5+51.3545s^4} \qquad (4.42)$$

$$\frac{+8.7835s^2+0.9760}{+42.1858s^3+28.0757s^2+11.5512s+2.9598}$$

The input admittance to the filter pair is calculated from (2.13) and is:

$$Y_{in} = \frac{1.9838s^8+11.5512s^7+19.2917s^6}{3.9358s^8+11.55512s^7+36.8596s^6} \qquad (4.43)$$

$$\frac{+42.1858s^5+35.6169s^4+42.1861s^3}{+42.1858s^5+67.0920s^4+42.1861s^3}$$

$$\frac{+19.2925s^2+11.5515s+1.9839}{+36.8595s^2+11.5515s+3.9357}$$

The input admittance can be separated into lowpass and highpass filter parts and the result is:

$$Y_{in} = \frac{0.4784s^3+0.1626s^2+0.4238s+0.0861}{s^4+0.6280s^3+1.4820s^2+0.5312s+0.2300} \qquad (4.44)$$

$$+ \frac{0.4571s^4+2.0653s^3+1.4281s^2+2.8880s}{s^4+2.3437s^3+6.8798s^2+3.6988s+6.2560}$$

$$+ \ 0.1303$$

The Darlington procedure and Cauer synthesis yield the realization shown in figure 4.10.

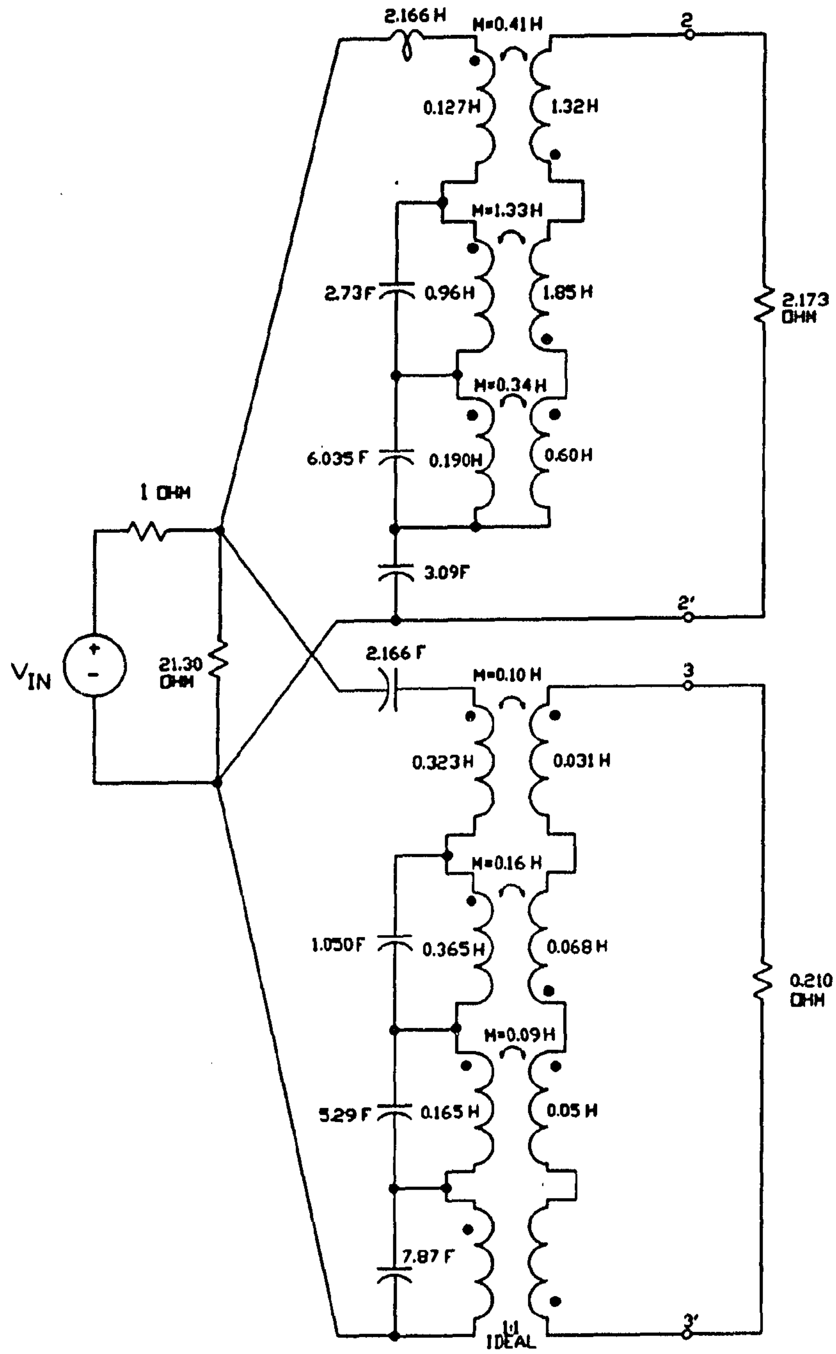

Figure 4.10. Fourth-order Chebyshev filter pair

D. Elliptic 3-port Filters

The elliptic filter pair requires the creation of Chebyshev rational functions from the Jacobian elliptic integral of the first kind. The creation of these functions requires the evaluation of the Jacobian elliptic sine and cosine functions. These rational functions are to be used as the approximation polynomials in the expansion of (4.21a) for the second- and third-order cases which will be evaluated in the following sections. The derivation of the rational functions will not be treated here. Chen (17) has a very good treatment of the origins of the Chebyshev rational functions and their use in elliptic filters. The numerical examples of the functions used in this research were taken from this source.

1. Second-order filter pairs

The Chebyshev rational function which will be used in the second-order filter pair example has a selectivity factor of 1.4. This is a measure of the steepness of the transition band between the passband and the stopband. Unlike the Butterworth and Chebyshev filters, the transition steepness is a variable parameter in the elliptic filter. The rational function, after evaluation of the elliptic sine and cosine functions, is:

$$F_2(\omega) = 1.30373 \frac{0.58824-\omega^2}{1-0.30012\omega^2} \tag{4.45}$$

This function is used in (4.21a) with a ripple factor which yields 0.5 dB ripple. The resulting reflection coefficient magnitude is:

$$S_{11}(s)S_{11}(-s) = \frac{0.0426s^8+0.1949s^6+0.3082s^4+0.1949s^2+0.0426}{0.9125s^8+3.3086s^6+5.5811s^4+3.3086s^2+0.9125} \tag{4.46}$$

The roots of (4.46) are listed in table 4.4. The minimum-phase reflection coefficient is constructed from the LHP poles and zeros in table 4.4:

$$S_{11}(s) = \frac{0.06832s^4+0.0006341s^3+0.1563s^2+0.0006341s+0.06832}{s^4+2.7384s^3+3.9187s^2+2.0522s+1} \tag{4.47}$$

The input admittance of the filter pair is calculated using (2.13), which results in:

$$Y_{in} = \frac{0.8900s^4+2.6152s^3+3.5939s^2+1.9597s+0.8900}{1.0205s^4+2.6165s^3+3.8925s^2+1.9609s+1.0205} \tag{4.48}$$

Table 4.4. Roots of Equation (4.45)

Root Number	Numerator Root Value	Denominator Root Value
1	+0.001720+j0.7670	+0.2284+j0.5936
2	+0.001720-j0.7670	+0.2284-j0.5936
3	-0.001720+j0.7670	-0.2284+j0.5936
4	-0.001720-j0.7670	-0.2284-j0.5936
5	+0.002923+j1.3038	+1.1408+j1.0818
6	+0.002923-j1.3038	+1.1408-j1.0818
7	-0.002923+j1.3038	-1.1408+j1.0818
8	-0.002923-j1.3038	-1.1408-j1.0818

This input admittance can be expanded into separate lowpass and highpass parts as follows:

$$Y_{in} = \frac{0.0562s+0.007642}{s^2+0.4265s+0.3995} + \frac{0.0191s^2+0.3115s}{s^2+2.1374s+2.5033} + 0.8530 \qquad (4.49)$$

Synthesis is carried out using the Darlington procedure with the Cauer synthesis, yielding the circuit shown in figure 4.11.

The circuit in figure 4.11 was simulated using the SPICE circuit analysis program. The frequency responses of the

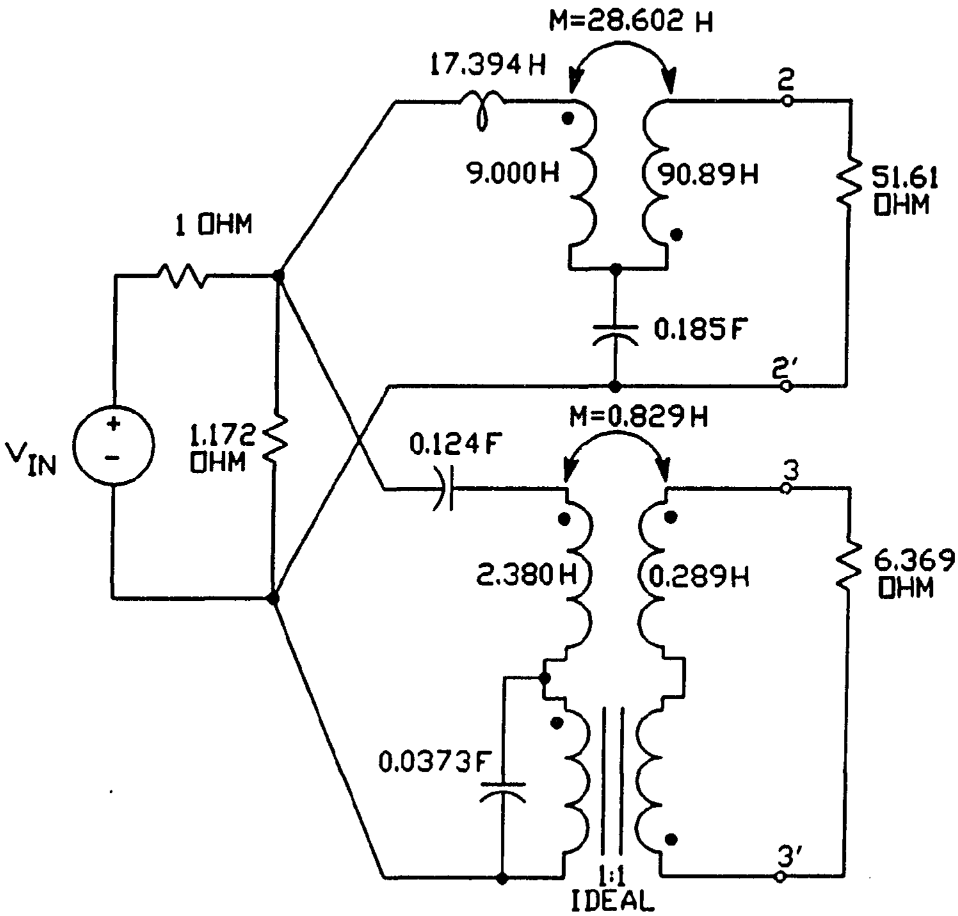

Figure 4.11   Second-order elliptic filter pair

lowpass and highpass filters as calculated by SPICE are shown in figure 4.12. As in the even-order Chebyshev case, the even-order elliptic response is at a minimum at the passband frequency extremes. The ripple, as in the Chebyshev case, is higher than the design value. The stopband of the elliptic response is devoid of ripples. This is because the synthesis of the network was carried out using the reflection coefficient as a source for the input admittance. The stopband ripple characteristic is present in the reflection coefficient, but very small in magnitude, and does not yield transmission zeros. The transmission coefficients of the elliptic filter pairs do contain stopband transmission zeros, however, and are realizable by use of the Brune and Gewertz or Bode methods (18). The preservation of the stopband ripples was not considered important to this research, and the Darlington synthesis was used to preserve the passband ripples.

2. Third-order filter pairs

The rational function chosen for use in the third-order example has the same parameters as in the previous example, and is represented as:

$$F_3(\omega) = 3.1163\,\omega\,\frac{0.81206-\omega^2}{1-0.4143\omega^2} \tag{4.50}$$

This function is used in (4.21a) with a ripple factor which yields 0.5 dB ripple to yield the following reflection coefficient magnitude:

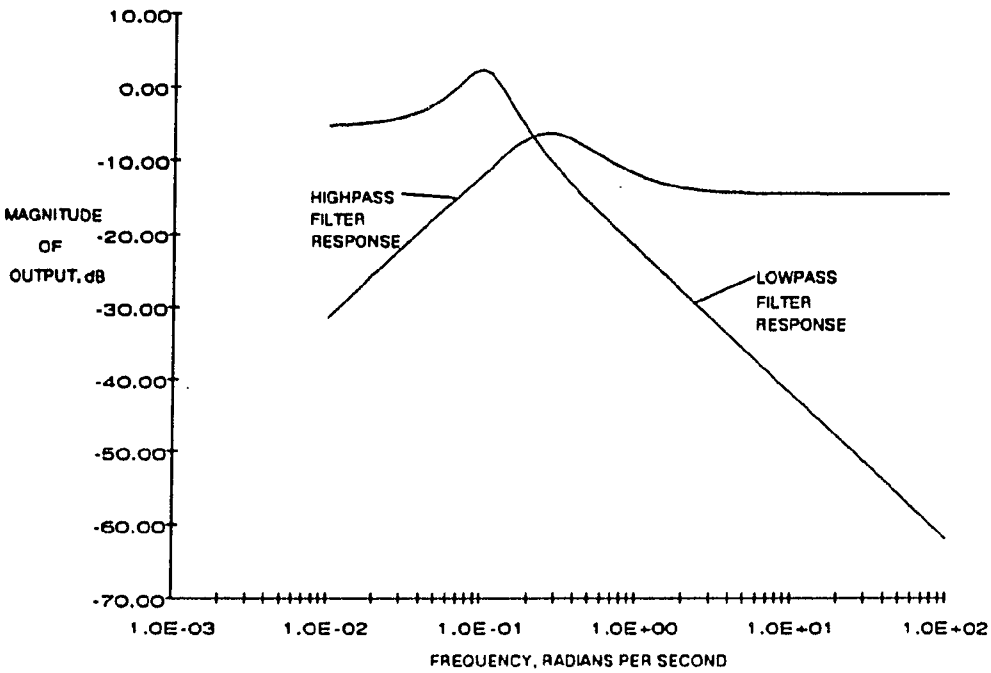

Figure 4.12. Frequency response of the second-order elliptic filter pair

$$S_{11}(s)S_{11}(-s) = \frac{-0.4436s^{10}-1.8131s^{8}-2.7398s^{6}-1.8131s^{4}-0.4436s^{2}}{0.5678s^{12}+0.8668s^{10}-0.9781s^{8}-2.6251s^{6}-0.9781s^{4}+0.8668s^{2}+0.5678} \quad (4.51)$$

The roots of (4.51) are listed in table 4.5. The minimum-phase reflection coefficient is constructed from the LHP poles and zeros in table 4.5:

Table 4.5. Roots of Equation (4.50)

Root Number	Numerator Root Value	Denominator Root Value
1	+0.005797+j0.1547	+0.1828+j0.9153
2	+0.005797-j0.1547	+0.1828-j0.9153
3	-0.005797+j0.1547	-0.1828+j0.9153
4	-0.005797-j0.1547	-0.1828-j0.9153
5	+0.007142+1.1099	+0.2098+j1.0507
6	+0.007142-1.1099	+0.2098-j1.0507
7	-0.007142+1.1099	-0.2098+j1.0507
8	-0.007142-1.1099	-0.2098-j1.0507
9	0	+0.8003
10	0	-0.8003
11		+1.2495
12		-1.2495

$$S_{11}(s) = \frac{0.6661s^5+0.0172s^4+1.36613s^3}{0.7535s^6+2.1363s^5+3.6035s^4+4.5390s^3} \quad \frac{+0.0172s^2+0.6660s}{+3.6034s^2+2.1363s+0.7535} \qquad (4.52)$$

The input admittance for this filter pair is calculated directly from (2.13) and is:

$$Y_{in}(s) = \frac{0.7535s^6+1.4703s^5+3.5862s^4+3.1778s^3+3.5862s^2+1.4702s+0.7635}{0.7535s^6+2.8023s^5+3.6207s^4+5.9003s^3+3.6207s^2+2.8023s+0.7535} \quad (4.53)$$

This input admittance can be expanded into separate lowpass and highpass parts as follows:

$$Y_{in}(s) = \frac{0.4391s^2+0.1628s+0.2484}{s^3+0.5851s^2+0.8744s+0.2790} + \frac{0.8904s^3+0.5834s^2+1.5737s}{s^3+3.1339s^2+2.0971s+3.5842} + 0.1096 \quad (4.54)$$

This input admittance can be synthesized using Darlington's procedure to obtain the y-parameters and using the Cauer synthesis to obtain the circuit structure. The resulting circuit is shown in figure 4.13.

The circuit in figure 4.13 was simulated using the SPICE circuit analysis program. Figure 4.14 shows the frequency responses of the lowpass and highpass filters calculated by SPICE. The responses are at maxima at the passband frequency extremes, as they should be. The ripple value is near the design value.

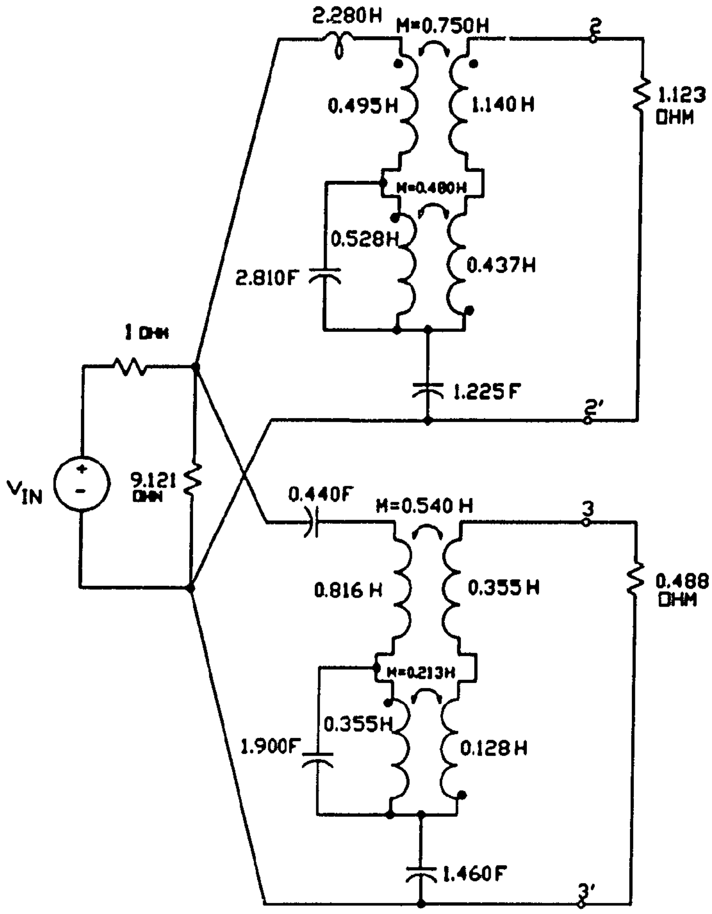

Figure 4.13. Third-order elliptic filter pair

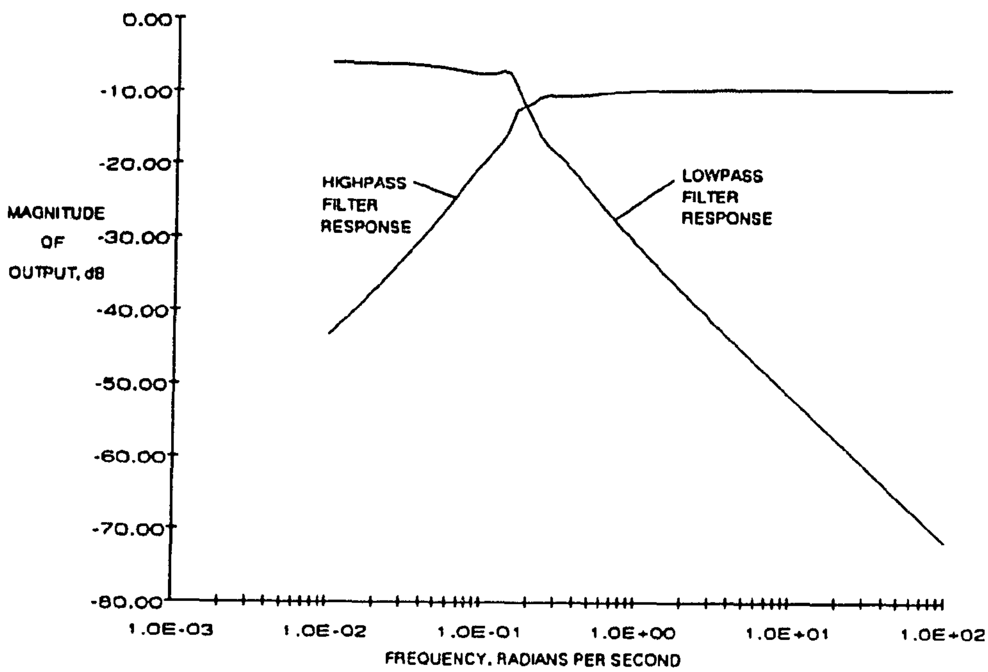

Figure 4.14. Frequency response of the third-order elliptic filter pair

V. SUMMARY AND CONCLUSIONS

A. Existence of Chebyshev and elliptic 3-Port Filters

The goal of this research was to find filter pairs which exhibit the Chebyshev and elliptic response characteristics. The main result of the research was the discovery of a set of reflection and transmission coefficients which satisfy the fundamental requirements for passive networks, and which exhibit response characteristics which approximate the Chebyshev and elliptic characteristics. The reflection coefficient was used to generate a positive-real input admittance for a resistively terminated network. The input admittance was shown to be always positive-real, and expandable into a shunt resistor in parallel with two new input admittances, one representing a lowpass filter, and the other a highpass filter. The new input admittances were shown to be positive real, and therefore capable of being transformed into real networks. Several examples were developed to show this. Odd and even order cases were shown to exist. The separation of the input admittance brought out a shunt resistance which represents the minimum real part of the input admittance. This resistor can be combined with the source resistor in a Thevenin equivalent circuit, but this technique was not used.

Also shown was a result that perfect Chebyshev and elliptic filters do not exist, but that perfect Butterworth filter pairs do exist. When the Butterworth approximating monomials are substituted into the defining equation for the reflection coefficient as derived in this research, the resulting filter pair has constant input resistance.

B. Realization of the Filters

Once the input admittances of the filters composing the filter pairs were derived, the lowpass and highpass filters were synthesized. In all cases presented, the synthesis was carried out using the Darlington procedure to derive the network admittance parameters, and the actual synthesis of the circuits was done using the Cauer synthesis technique. The separation of the input admittance into separate lowpass and highpass filter sections was shown to require a shunt resistance which dissipated some power from the input. The value of this shunt resistor increased rapidly with filter order, however, and was almost negligible in the fourth-order case.

VI. SUGGESTIONS FOR FURTHER STUDY

A. Non-complementary Filter Pairs

This research was limited to filter pairs which are complementary, a case in which the lowpass is a function of s, while the corresponding highpass filter is a function of 1/s. In point of fact, the scattering parameters in (4.21) are not strictly dependent on this relationship, and will work with any set of approximation functions. This is true because the nature of the approximation function is not what causes (4.20) to balance. Further study could investigate the effects of other approximation functions.

The research was also limited to the case in which the filters in the filter pair are of the same order. Unequal orders in the lowpass and highpass can be explored. This case is easily investigated by using approximation polynomials of different order.

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I dedicate this dissertation to four people:

To my wife, Astrid, and to my daughter, Leslie, for their support in this most arduous and rewarding endeavor.

To the memories of my father, Robert, and his father, Wallace, for their encouragement and guidance during their living years. I have gone where they wanted to go.